LONDON'S EQUATIONS

According to London's theory, there are two types of conduction electrons in a superconductor namely the superelectrons and the normal electrons.

At 0k a superconductor contains only superconducting electrons, but as the temperature increases the ratio of the normal electrons to superconducting electrons

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(10)

$\therefore \quad J = e n_n V_n + e n_s V_s \longrightarrow (II)$

In a superconducting metal, the superconducting electron encounter no resistance to their motion so,

So, The equation of the motion of superconducting electron is

$$ma = eE$$ where E = Applied field.

$$m \cdot \frac{dV_s}{dt} = eE$$

$$m \dot{V}_s = eE \quad \text{or} \quad \dot{V}_s = \frac{eE}{m}$$

The supercurrent density is

$$J_s = e n_s V_s$$

or $$\frac{dJ_s}{dt} = e n_s \frac{dV_s}{dt}$$

On putting the value of $\dot{V}_s$ in above eqn we get

$$\frac{dJ_s}{dt} = e n_s \cdot \frac{eE}{m}$$

$$\boxed{\dot{J}_s = \frac{n_s e^2 E}{m}}$$

This is the LONDON's first equation which describes the absence of resistance.

Similarly The equation of normal current is

$$\boxed{J_n = \sigma_n E = \frac{n_n e^2 \tau E}{m}}$$ where

τ = Relaxation time

(ii)

From above eqn we observe that an electric field is necessary for established a steady current. If $E=0$, J_n becomes 0.

Now,

Using maxwells eqn.

$$\dot{B} = -\text{curl} E = -\mu \dot{H} \quad \text{—(II)}$$

where

$\dot{B}$ = magnetic field

H = magnetic field intensity.

Again from Maxwells equation

$$\text{curl} H = J_s + \dot{D} \quad \text{—(III)}$$

Now, taking curl of $\dot{B}$ we get.

$$\nabla \times \dot{B} = \nabla \times (-\mu \dot{H})$$

$$\nabla \times \dot{B} = -\mu_0 (\nabla \times \dot{H})$$

$$\nabla \times \dot{B} = -\mu_0 (J_s + \dot{D}) \quad \text{from eqn (III)}$$

$$\nabla \times \dot{B} = -\mu_0 (J_s + \dot{D}) \quad \text{—(IV)}$$

When the ~~electric~~ electric field varying rapidly with time, the displacement current $\dot{D}$ is negligible in comparision with to J_s.

so, put $\dot{D} = 0$

Then eqn (II) and (IV) take the form.

$$\text{curl} H = J_s$$

$$\dot{B} = -\text{curl} E \quad \text{—(V)}$$

and

$$\nabla \times \dot{B} = +\mu_0 (J_s + 0)$$

$$= +\mu_0 J_s \quad \text{—(VI)}$$

(12)

$\therefore$ From eqn, $J_s = \frac{n_s e^2 E}{m}$

or, $J_s m = n_s e^2 E$

On taking curl we get

$m(\nabla \times J_s) = n_s e^2 (\nabla \times E)$

$\therefore \text{curl}\, E = \frac{m \,\text{curl}\, J_s}{n_s e^2}$

On putting the value of curl E in eqn (V) we get

$\dot{B} = -\text{curl}\, E$

$= -\frac{m}{n_s e^2}(\text{curl}\, J_s)$ — (VII)

$\therefore$ From eqn (VI)

$\nabla \times \dot{B} = \mu_0 J_s$

$\cancel{J_s = \mu_0 J_s}$.

$\therefore J_s = \frac{\nabla \times \dot{B}}{\mu_0}$

$= \frac{\text{curl}\, \dot{B}}{\mu_0}$

On putting J_s in above eqn (VII) we get

$\dot{B} = \frac{-m}{n e^2} \frac{(\text{curl}\, \dot{B} \cdot \text{curl}\, \dot{B})}{\mu_0}$

$= \frac{-m}{n e^2}$ $\cancel{\text{curl}\, \dot{B}\, \text{curl}}$ curl curl $\dot{B}$

$\dot{B} = -\alpha \cdot \text{curl} \cdot \text{curl}\, \dot{B}$ — (VIII) [put $\alpha = \frac{m}{e^2 n}$]

∴ we know that

$$\nabla \times \nabla \times \vec{B} = \text{grad div } \vec{B} - \nabla^2 \vec{B}$$

$$= \text{grad } \nabla \cdot \vec{B} - \nabla^2 B$$

But from maxwell's eqn

$$\nabla \cdot B = 0.$$

Then,

$$\nabla \times \nabla \times \vec{B} = 0 - \nabla^2 \vec{B}$$

$$\nabla \times \nabla \times \vec{B} = - \nabla^2 \vec{B}$$

On putting the value of ~~$\nabla\times\nabla\times\vec{B}$~~

$$\vec{B} = - \alpha \cdot \nabla \times \nabla \times \vec{B}$$

On substituting this in eq above equation we get

$$\nabla \times \nabla \times \vec{B} = - \nabla^2 \left(- \alpha \cdot \nabla \times \nabla \times \vec{B} \right)$$

$$\cancel{\nabla \times \nabla \times \vec{B} = - \alpha \nabla^2 \left(\nabla \times \nabla \times \vec{B} \right)}$$

$$\vec{B} = + \alpha \nabla^2 \vec{B}$$

or $\boxed{\nabla^2 \vec{B} = \frac{\vec{B}}{\alpha}}$

F. London and H. London suggested that the magnetic behaviour of superconducting metal can also be described by the following expression

$$\nabla^2 B = \frac{B}{\alpha}$$

The solution of the above equation is

$$B(x) = B_0 \exp\left(-x/\sqrt{\alpha}\right).$$